

## Diagonalize

Employs the Jacobi method to compute the eigenvectors and eigenvalues of a real symmetric matrix. This tool can be used to compute the molecular orbital coefficients via simple Huckel MO theory. In this method, the real symmetric matrix is created by entering unity for element  $a_{ij}$  if atom  $i$  is bonded to atom  $j$  or zero if the atoms are not bonded.

### Input range

Type the reference for the range of the real symmetric matrix. This range must be square ( $N \times N$ ). Since only the top half is used, the user need not define elements below the diagonal.

### Output range

Type the reference for the upper-left cell of the output range. The output will be written in an  $(N+1 \times N)$  range. The  $N$  eigenvectors are returned in  $N$  columns (of  $N$  rows) and the last row contains the eigenvalues.

## CustomFit

In data fitting, one typically has  $m$  data values  $y_1, y_2, \dots, y_m$  which have been sampled for values  $x_1, x_2, \dots, x_m$  of some independent variable  $x$ . It is then desired to fit a function  $f(x, p)$  which has  $n$  adjustable parameters, to be chosen so that the function best fits the data. The residuals are given by

$$r_i(p) = f(x_i, p) - y_i \quad i = 1, 2, \dots, m$$

and a least squares solution is sought by minimizing  $S(p)$  which is the sum of the squares of the residuals. The Levenberg-Marquardt method attempts to solve for the solution to  $S(p)$  by stepping towards the solution via a sequence of corrections based on the solution to the equation

$$(\mathbf{G} + a\mathbf{I})\mathbf{s} = -\mathbf{g}$$

where  $\mathbf{G}$  is a matrix of second derivatives,  $\mathbf{g}$  is a matrix of first derivatives,  $\mathbf{I}$  is a unit matrix and  $a$  is a damping factor. The Levenberg-Marquardt method is also characterized by the scaling of the matrix  $\mathbf{G}$  by replacing it with a scaled matrix  $\mathbf{C}$  where

$$C_{ij} = G_{ij} / ((G_{ii})^{1/2} \cdot (G_{jj})^{1/2}) \quad \text{or} \quad \mathbf{C} = \mathbf{D}^{-1}\mathbf{G}\mathbf{D}^{-1}$$

The solution for the step  $\mathbf{s}$  is then given by

$$\mathbf{s} = -\mathbf{D}^{-1}(\mathbf{G} + a\mathbf{I})^{-1}\mathbf{D}^{-1}\mathbf{g}$$

where  $\mathbf{D}^{-1}$  is a diagonal matrix and  $(\mathbf{G} + a\mathbf{I})^{-1}$  is determined from the eigenvalue decomposition as  $\mathbf{C} = \mathbf{A}\mathbf{L}\mathbf{A}^T$  where  $\mathbf{L}$  is a diagonal matrix of eigenvalues and  $\mathbf{A}$  is a matrix of eigenvectors. If none of the eigenvalues are zero, then the inverse of  $\mathbf{C}$  exists and is defined as  $\mathbf{C}^{-1} = \mathbf{A}\mathbf{L}^{-1}\mathbf{A}^T$ .

Pragmatically, iterations must also be halted when no significant change is obtained in two successive choices of the parameters. This condition will be encountered when the model nonlinear equation has a parameter redundancy. For reference see Marquardt, D.M. "An algorithm for Least-Squares Estimation of Nonlinear Parameters", J. Soc. Indust. Appl. Math., 11, 431-441, 1963

As a general rule, "best-fitting" is obtained only when

1. termination occurs before the maximum number of iterations (50)
2.  $S$  is minimal
3. there is no parameter redundancy indicated by the eigenvalues near zero
4. scale vectors are finite

To use CustomFit, the user must supply a user written macro as illustrated below for the function  $y = ax/(b+x)$ .

```
USERMACRO
=RESULT(1)
=ARGUMENT("kIndex",1)
=ARGUMENT("rgP",64)
=ARGUMENT("kX",1)
=SET.VALUE(D20:E20,rgP)
Ret2
=D20*nX/(E20+kX)
=RETURN(Ret2)
```

The macro takes three arguments,  $kIndex$  - the index of the  $k$ 'th data point;  $rgP$  - the array of parameters;  $kX$  - the value of the  $k$ 'th independent variable. The macro must return a value for the calculated  $y$  (dependent variable). In addition, the name "CustomFitMacro" must be defined as "USERMACRO" in the worksheet. In this example, the user would select the define names command and define the name "CustomFitMacro" as "=MACRO1.XLM!USERMACRO". The name USERMACRO must also be a defined name in the macro sheet. The index  $k$  is not normally required but may be used if direct indexing of the dependent variable is required or preferred.

### Data range

Type the reference for the range of the independent and dependent variables. This range must be a single continuous range of  $m$  rows by 2 columns with the independent variable ( $X$ ) in the first column.

### Parameter range

Type the reference for the range of the parameters and their upper and lower bounds. This range

must be a single continuous range of 3 rows and n columns. The first row contains the initial starting parameters, the second row the upper bound for these parameters and the third row is the lower bound for these parameters.

#### **Output range**

Type the reference for the upper-left cell of the output range. The output will be written in an  $m \times (n+1)$  range. The first column contains the fitted Y values. The other columns contain additional information and are as follows:

final parameters ( $p_1, p_2, \dots, p_n$ )

standard deviation of the final parameter estimate (use to construct confidence intervals)

eigenvalues (non zero if no redundancy)

scale vectors (measure of response to change in parameter value).

#### **Hint:**

If you experience difficulties in getting any results, check your macro and in particular, verify that your macro is receiving and using all of the required parameters.

## PolyCurveFit

Polynomial curve fitting results in a single polynomial equation of order  $m$  which is the least squares approximation of the observed data.

$$y = c_0 + c_1 * X + c_2 * X^2 + c_3 * X^3 \dots + c_m * X^m$$

This tool will compute the coefficients  $c_i$  for the polynomial which minimizes the sum of the squares of the deviations from the calculated and observed values for  $y$ . In addition to the coefficients, this command will return values which enable the user to assess the quality of the fit.

The most important measure of the quality of the polynomial fit is the correlation coefficient. The closer this value is to 1, the better the fit. Another measure of the fit is the coefficient of determination, usually referred to as  $R^2$ . This value is equal to the square of the correlation coefficient. The standard error of the estimate, abbreviated as SEE, is a measure of the scatter of the actual data along the fitted line. The smaller the SEE value, the closer the actual data is to the computed polynomial.

## Input

### Xvar range

Type a reference to a column or row range for the  $N$  independent variables ( $X$ ).

### Yvar range

Type a reference to a column or row range for the  $N$  dependent variables ( $Y$ ).

$Xvar$  and  $Yvar$  must both be either row or column vectors. Using one as a row vector and the other as a column vector is not supported. It is recommended that both variables be entered as column vectors.

### Order

Type a number for the order  $m$  of the fitting, i.e. 1 for a linear fit, 2 for a quadratic fit and etc. The order must be one less than the number of variables.

## Output

Type the reference for the upper-left cell of the output range. The output range is an  $N \times 3$  array if the input is in the form of a column vector or a  $3 \times N$  array if the input is in the form of a row vector.

Assuming that both  $Xvar$  and  $Yvar$  are column vectors and the array formulae have been entered into a  $N \times 3$  array, then the first column of the output contains the estimated  $Y$  values, the second column contains the residuals (differences between calculated and estimated  $y$ -values). The third column contains in the first  $(order + 1)$  rows, the polynomial coefficients. If fitted to 2nd order, the first three rows contain  $c_0$ ,  $c_1$ , &  $c_2$ . The following values are returned directly below the coefficients, *coefsig* - a vector of dimension  $(order+1)$ . *Coefsig* are the standard errors of coefficient estimates.

The values are stored in the same order as the polynomial coefficients.

*see* - the standard error of the estimate

*rsqrval* - the  $r$  squared value - the sample correlation coefficient

*cferror* - returns 1 if the curve fit is singular, otherwise 0.

## CubicSplines

Fits a discrete set of cubic polynomial equations to a discrete set of data points. Whereas polynomial curve fitting produces a single equation to fit the data points, cubic splines curve fitting produces a family of cubic equations, one cubic equation for each interval in the original data. Cubic spline curve fitting guarantees that the fitted curve will pass exactly through the original data points.

The original X values and returned cubic spline coefficients may be subsequently used to interpolate for points between the original data points. For details see [CalcSpline](#).

## Input

### Xvar range

Type the reference for a range of N independent variables (X).

### Yvar range

Type the reference for a range of N dependent variables (Y).

Both variables can be entered as row or column vectors.

## Output

Type the reference for the upper-left cell of the output range. CubicSplines will ALWAYS return the coefficients in an Nx4 range.

## SmoothSG

Performs a Savitsky - Golay simplified least squares smoothing and differentiation of data (see Savitsky, A. and Golay, J., Analytical Chemistry 36 (1964), p. 1627). This technique uses convolution where each data point is recalculated as a weighted average of its original value and the surrounding data points. The degree of smoothing is a function of the number of surrounding data points used in the convolution, and the larger the convolution kernel, the larger the degree of smoothing.

## Input

### Data range

Type a reference to a column or row vector of the data to be smoothed. If the data is a column vector, then the output is a column vector and *vice versa* if the data is a row vector.

### SmoothNum -

Type the integer degree of smoothing

1 = 5 point smooth

2 = 7 point smooth

3 = 9 point smooth

4 = 11 point smooth

5 = 13 point smooth

### DerivNum

Type the integer derivative degree

0 = smooth data only

1 = first derivative

2 = second derivative

## Output

Type the reference for the upper-left cell of the output range. If the input is entered in a column range, the output will return in a column range. If the input is entered as a row range, the output will return as a row range.

## SmoothWT

Is used to reduce the noise in a sample. The technique uses convolution where each data point is recalculated as a weighted average of its original value and surrounding data points. The degree of smoothing is a function of the number of surrounding data points used in the convolution and the user supplied weights used in the recalculation.

## Input

### Data range

Type a reference for the range of data be smoothed. The data may be entered into a column or row vector.

### Weights range

Type a reference for the range of weights used in the convolution process

## Output

Type the reference for the upper-left cell of the output range. If the input is entered in a column range, the output will return in a column range. If the input is entered as a row range, the output will return as a row range.

**Exit**

Select this command to remove the XLMath add-in tools.



## MODensity

Will allow the user to calculate the pi atom charge and bond order matrix for a simple Huckel calculation. The charge and bond order matrix in simple Huckel calculations is defined as a matrix multiplication of  $C^T \times Occ \times C$  where  $C$  is the matrix of coefficients and  $Occ$  is a 1 dimensional matrix of occupancies. You must calculate the molecular orbital coefficients prior to calculating the charge and bond orders,

## Input

### Coef range

Type a reference for the range of the Coefficients(*Coef*).

### Occ range

Type a reference for the range of Occupancies(*Occ*).

## Output

Type the reference for the upper-left cell of the output range. The output will be returned in an NxN range. In the output matrix, the diagonal elements represent the charges on the atoms and the off-diagonal elements between two bonded atoms represent the pi bond orders. Off-diagonal elements between two non bonding atoms have no meaning.

## CalcSpline

Will calculate the cubic spline interpolated Y value of a given X value. Y values can be interpolated between the first and last original X values used to calculate the cubic spline coefficients. CalcSpline cannot be used to interpolate beyond the end points of the original X values. By definition, X values identical to those used to calculate the coefficients, will have an interpolated Y value identical to the original Y value.

## Input

### Xorig range

Type a reference for the original set of data points used to calculate the cubic spline coefficients.

### Coef range

Type a reference for the range of coefficients previously calculated in [CubicSplines](#).

### Xcalc range

Type a reference for the range of independent variables (X) for which you wish to calculate interpolated Y values. NOTE, this is not the range used to calculate the coefficients but a new range of X values for which interpolated Y values are required.

## Output

Type the reference for the upper-left-cell of the output range. The output will be returned in a column vector if Xorig is a column vector and in a row vector if Xorig is a row vector.

## **XLMATH Command (Main Menu)**

Displays the Xlmath menu with a list of curve fitting and Huckel molecular orbital tools.

### **Xlmath**

Lists the available Xlmath tools.

### **See Also**

Using an Xlmath tool.

<u>Diagonalize</u>	Diagonalize a real symmetric matrix.
<u>MODensity</u>	Calculate charges & bond orders
<u>Polynomial</u>	Polynomial fitting of data points
<u>Cubicspline</u>	Cubic spline function fitting
<u>CustomFit</u>	Fitting to a user defined nonlinear function
<u>CalcSpline</u>	Interpolate between data points
<u>SmoothSG</u>	Savitsky-Golay smoothing of data.
<u>SmoothWt</u>	Data smoothing via weighted convolution
<u>Exit</u>	Remove Xlmath add-in tools
<u>Custom Functions</u>	View help on custom functions

## **Xlmath Custom Functions**

All of the commands in the Xlmath menu have corresponding custom functions. These custom functions can be accessed through the Excel Paste Function command and are listed under the heading Xlmath Add-In. All of the custom functions return a single array which is identical in form to the output returned in the commands. This means that the custom function with the appropriate arguments must be entered into each cell of the output array. If you are not familiar with Excel's array formula usage, please read the section on array formulae in the Excel User's Guide. Custom functions are useful if your input data changes frequently. In this case, entering the custom functions in an array formula will allow for the automatic updating of the output.

The following is a description of each custom function in Xlmath Add-In

**Function: PolyCurveFit(Xvar, Yvar, Order)**

Arguments: Xvar - the range for the dependent variables  
Yvar - the range for the dependent variables  
Order - the order of the fit

**Function: CubicSplines(Xvar, Yvar)**

Arguments: Xvar - the range for the dependent variables  
Yvar - the range for the dependent variables

**Function CalcSpline(Xorig, Coef, Xcalc)**

Arguments Xorig - the range for the original data points  
Coef - the range for the coefficients derived from CubicSplines()  
Xcalc - the range for the X values for which Y values will be calculated

**Function: SmoothSG(Data, SmoothNum, DerivNum)**

Arguments: Data - the range for the data points  
SmoothNum - a reference to the degree of smoothing  
DerivNum - a reference for the derivative degree

**Function: SmoothWT(Data, Weights)**

Arguments: Data - the range for the data points  
Weights - the range for the convolution weights

**Function: Diagonalize(SymMat)**

Arguments: SymMat - the range for the real symmetric matrix

**Function: MODensity(Coef, Occ)**

Arguments: Coef - the range for the coefficients derived from diagonalize  
Occ - the range for the orbital occupancies

### **See Also**

<a href="#"><u>Diagonalize</u></a>	Diagonalize a real symmetric matrix.
<a href="#"><u>MODensity</u></a>	Calculate charges & bond orders
<a href="#"><u>Polynomial</u></a>	Polynomial fitting of data points
<a href="#"><u>CubicSpline</u></a>	Cubic spline function fitting
<a href="#"><u>CalcSpline</u></a>	Interpolate between data points
<a href="#"><u>CustomFit</u></a>	Fitting to a user defined nonlinear function
<a href="#"><u>SmoothSG</u></a>	Savitsky-Golay smoothing of data.
<a href="#"><u>SmoothWts</u></a>	Data smoothing via convolution

## Using Xlmath

Before you use an Xlmath tool, you must organize your data into columns or rows on a worksheet. This is your input range. If you label your variables with text labels, do not include these labels in your input range. All of your data must be in continuous ranges.

There are two alternative ways of using Xlmath tools. The first way is to select the tools in the Xlmath menu. The second way is to enter Xlmath functions directly into array formulae.

### Xlmath Menu Tools

When you use an Xlmath menu tool, Excel creates an output table of the results. The contents of the table depend upon the tool that you are using. As a general rule, if your input range is a column vector, a  $N \times 1$  range, Excel will write the output table in a column vector. If your input range is a row vector, a  $1 \times N$  range, Excel will write the output table in a row vector. An exception to this rule can be found in the CubicSplines tool where the output is always a  $N \times 3$  range.

1. From the main menu choose Xlmath.
2. In the Xlmath box, select the tool that you want to use.
3. Type the input range, the output range and any requested numerical input.  
You can type cell ranges in boxes by typing a cell or range reference or a name of a cell or range reference, or by selecting the cell range on the worksheet.
4. Choose the OK button.

### Restriction

When you are prompted in a dialog box for an input range, you must enter a range of at least two cells. Entering a reference to a single cell will halt the tool and produce an error message.

### Xlmath Custom Functions

The Xlmath custom functions can be chosen from the Excel Paste Function menu. The custom functions are listed under the category Xlmath Add-In and must be pasted into an array formula which has the exact dimensions of the output table returned by the Xlmath menu tools.

### See Also

#### User's Guide (Book1)

<a href="#"><u>Diagonalize</u></a>	Diagonalize a real symmetric matrix.
<a href="#"><u>MODensity</u></a>	Calculate charges & bond orders
<a href="#"><u>Polynomial</u></a>	Polynomial fitting of data points
<a href="#"><u>Cubicspline</u></a>	Cubic spline function fitting
<a href="#"><u>CalcSpline</u></a>	Interpolate between data points
<a href="#"><u>CustomFit</u></a>	Fitting to a user defined nonlinear function
<a href="#"><u>SmoothSG</u></a>	Savitsky-Golay smoothing of data.
<a href="#"><u>SmoothWt</u></a>	Data smoothing via weighted convolution
<a href="#"><u>Exit</u></a>	Remove Xlmath add-in tools
<a href="#"><u>Custom Functions</u></a>	View help on custom functions

Chapter 5, "Creating a Worksheet: Using Array Functions"

